During the course of these measurements, the work was done with different departures with respect to the divisions, in order to show up the irregularities of the microscopes if such existed.

Origin.	*Angle.*	*Discrepancy.*
0	10^G 03 49.2	1.5
10	47.8	2.9
20	46.8	3.9
30	49.4	1.3
40	49.7	1.0
50	50.5	0.2
60	48.6	2.1
70	54.1	4.6
80	53.4	3.3
90	52.4	1.7
100	55.8	5.1
110	47.1	3.6
120	51.4	0.7
130	45.8	4.9
140	55.4	4.7
150	48.4	2.3
160	54.7	4.0
170	47.4	3.3
180	54.4	3.7
190	52.4	1.7

Value of the angle : 10^G 03 50.7

AN OPTICAL DEVICE TO AID IN MAPPING FROM PHOTOGRAPHS

by

O. M. MILLER, AMERICAN GEOGRAPHICAL SOCIETY

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The operation of the single eyepiece oblique plotter which is being used at the American Geographical Society (1) enables level features, such as coastlines, to be drawn directly in plan from their perspective images appearing on high oblique photographs, that is to say, those taken with the camera axis pointing more nearly horizontal than vertical. Where the ground is not flat, the instrument can be operated to obtain the positions of points both in plan and in elevation by plotting the intersection of corresponding perspective rays from two photographs.

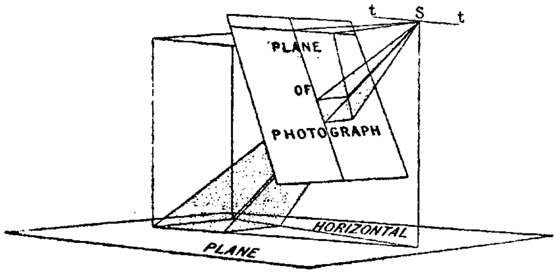

Fig. 1

(1) *The instrument has been built by the Mann Instrument Company.*

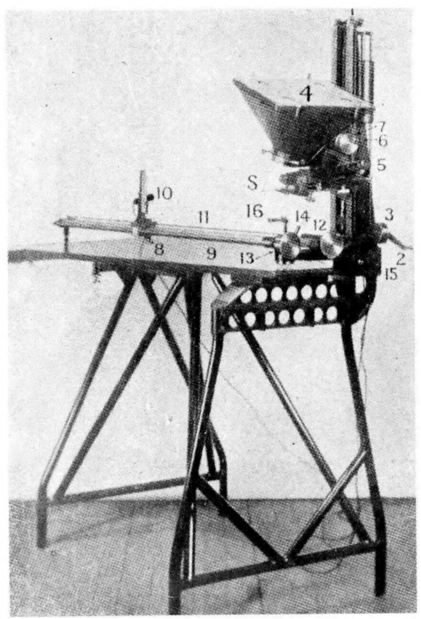

Fig. 4.

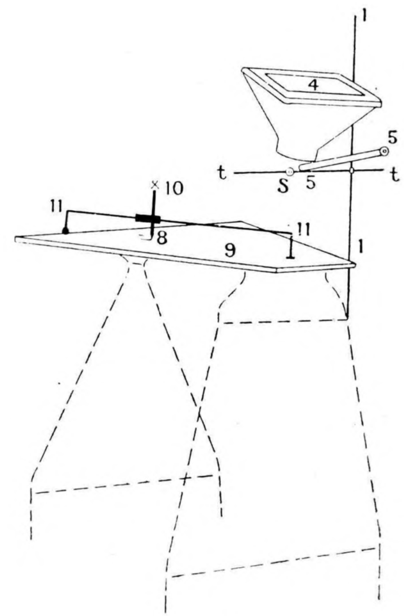

Fig. 5.

Fig. 1 is a perspective diagram which shows the geometrical relationship between the horizontal plane, the tilted photograph in its positive form and the perspective centre *S*, at the moment of exposure. *S* corresponds to the back nodal point of the camera lens as far as the photograph is concerned, the front nodal point as far as the ground or map is concerned, and the virtual position of the observer's eye as far as a plotting mechanism is concerned. When the photograph is placed in its correct relative orientation to the map or ground plane, then a line between any point on the ground and its image on the photograph if produced must pass through *S*. If the ground is flat, then this line can also be considered to pass through the map position of the point (see Fig. 2). If the ground is not flat, then the perspective ray between any point above the ground datum level and *S* will not pass through the map position of the point (see Fig. 3).

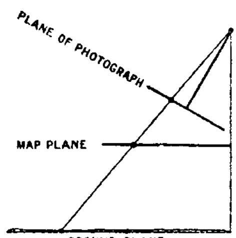

Fig. 2

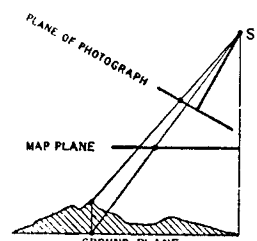

Fig. 3

However, the perspective centre, the photographic image of the point and the map position of the point all lie in the same vertical plane, so that though the actual map position of the point cannot be determined from one photograph its direction from the camera position can be obtained.

As far as flat country is concerned, Fig. 2 immediately suggests a simple though crude plotting apparatus. By using a positive transparency and by placing the observer's eye behind a small aperture at *S*, the image of the point of a plotting pencil moving in the map plane will be seen superimposed on the photograph. By moving the pencil under the features on the photograph, their plan outline can be plotted roughly. If the aperture is made small enough virtually to eliminate parallax errors and focusing difficulties, problems of illumination immediately arise.

E. Deville (1), the late Surveyor-General of Canada, in proposing a form of stereoscopic plotter, suggested placing half-silvered mirrors in front of two apertures, one for each eye. In this proposal an index mark attached to a plotting pencil was observed directly and the photographs were observed by reflection. The use of half-silvered mirrors, however, merely extended the use of the device to stereoscopic measurement but did not solve the problems of focus and illumination (2).

One solution of these problems is the use of what may be called a pinhole mirror, and this device has been incorporated into the oblique plotter whose main features will now be described briefly.

Fig. 4 is a photograph and Fig. 5 is a diagrammatic outline of the oblique plotter as at present constructed. A horizontal axis *t-t* is mounted on the vertical standard *1* so that it can be moved up and down the latter, by turning the crank *2*, the height of the axis being read on a drum *3*. *t-t* is the tilt axis about which the pinhole mirror *S*, a photograph holder *4* in which a negative is placed, and a viewing apparatus *5* can be rotated together by turning the crank *6*, the tilt being read on the drum *7*. The mirror consists of a fully reflecting plane surface in the centre of which is a pinhole aperture about one-third of a millimetre in diameter. The mirror is adjusted so that the tilt axis lies in its plane and the pinhole is centred on the tilt axis. The photograph holder is adjusted so that the plate perpendicular corresponding to the

(1) *E. Deville*, On the Use of the Wheatstone Stereoscope in Photographic Surveying, *Trans. Roy. Soc. Canada 8, 63 (1902).*

(2) *For brief descriptions of other developments in plotting instruments in which real index marks are used, see O. von Gruber,* Photogrammetry : Collected Lectures and Essays, *translated from the German original by G. T. McCaw and F. A. Cazalet, American Photographic Publishing Co., Boston, 1932, pp. 176-181, 221-223.*

optical axis of the camera cuts the tilt axis normally at the pinhole. Slow motion movements are provided for swinging the photograph in its own plane about the plate perpendicular and for setting the photograph at its equivalent focal distance from the pinhole, and vernier scales are provided for reading the amounts of these movements.

The inclination of the plate perpendicular to the plane of the mirror is actually about 60°, though theoretically the instrument would function at other inclinations also. The important thing to know, however, is the inclination of the reflected ray to the horizontal map plane as this corresponds to the angle of depression or elevation.

The viewing apparatus *5* consists of a periscope whose optical axis lies in the tilt plane and always points to the pinhole. This part of the instrument will be described more fully later.

Attached to and vertically above a plotting pencil *8* which, when operating, touches the surface of the plotting table *9*, is an index mark *10* consisting of a small aperture illuminated from behind through a condenser lens. The index mark and pencil are attached to a carriage which is free to move along the horizontal bar *11*. The bar *11* is free to swing around the vertical axis passing through the pinhole. By turning the crank *12* the bar is made to swing and a drum is attached which reads changes in horizontal direction. By turning the crank *13*, the pencil and index mark carriage are made to move along the bar, and a drum is attached which reads distances out from the centre of rotation. The pencil can be lifted off the plotting table by means of a bar lever operated at *14*. The intensity of the illumination of the index mark is controlled by a rheostat situated at *15* and it has been found convenient to make the colour of the illumination green or red. The angular scales on the instrument read to the nearest minute and the linear scales all read to the nearest 0.1 of a mm.

It will be apparent from the above description that the pinhole corresponds to the perspective centre *S* of Fig. 1 and that very thin pencils of rays from the index mark on passing through the pinhole are enveloped by corresponding pencils of rays from point images in the photograph. The effect to the observer looking through the periscope is to see the image of the index mark superimposed on the photograph. The periscope functions solely as a viewing apparatus whereby the eye is placed virtually at the pinhole. Theoretically, the pencils of rays from the point images in the photograph should be made parallel before being reflected from the pinhole mirror by means of an auxiliary lens placed between the photograph and the pinhole, but it has been found when this is not used that by limiting the size of the mirror surface and by using a low power viewing apparatus, good definition and sufficient illumination of the photograph can be maintained without introducing apparent parallax. The plateholder, however, is easily detachable from the rest of the instrument and an auxiliary lens could be placed in position without difficulty.

With the periscope focused on the photograph the index mark can be seen in sharp definition over a wide range, actually, on the instrument built, from less than 10 cm. to a metre out from the pinhole.

A further advantage of using an illuminated index mark and a pinhole mirror is that other objects behind the mirror not illuminated as brightly as the index mark cannot be seen, with the result that the image of the photograph is not confused.

As regards operating performance the preliminary tests of the instrument indicate that it is capable of plotting and recording directions correctly to, at any rate, the nearest two minutes of arc. When further refinements have been introduced it is hoped to be able to better this considerably.

The operation of the instrument is simple. To plot planimetry once the photograph has been correctly oriented, the operator merely moves the index mark so that its image follows the outline of the feature on the photograph. The scale of the drawing is determined by the ratio between the height of the pinhole above the map and the height of the camera station above the ground. To intersect a point above the map datum level it is necessary to plot first the relative horizontal positions of the two photographs on a plotting sheet and the directions in which the camera was pointing at the moment of exposure. The first photograph is taken and correctly oriented, and the sheet is then oriented on the board so that the direction of the plotted camera axis points at right angles to the tilt axis and the position of the camera station lies directly under the pinhole. To facilitate the latter a centring device *16* has been provided. The image of the index mark is then made to superimpose the image of the point to be intersected, the pencil is placed in operation and a line drawn on the paper by moving the index mark in towards the operator. The same process is repeated with the second photograph, and where the two lines intersect on the paper is the position of the point on the map. Then, by placing the pencil at this intersection and lowering the tilt axis until the image of the index mark again covers the photographic image of the point, the difference of height can be read directly on the height scale, the distance can also be read on the distance scale and the necessary correction to the height for curvature and refraction can be applied at once from tables.

The field of view of the periscope of the oblique plotter as constructed is sufficient to cover the field of a 7 × 9 inch photograph taken with a camera having a focal length of 10 inches, but only in an up and down direction. It is not sufficient to cover it laterally. To do this without moving the eye while plotting, a simple optical linkage has been incorporated into the instrument; the following is a brief description of the geometry of this device.

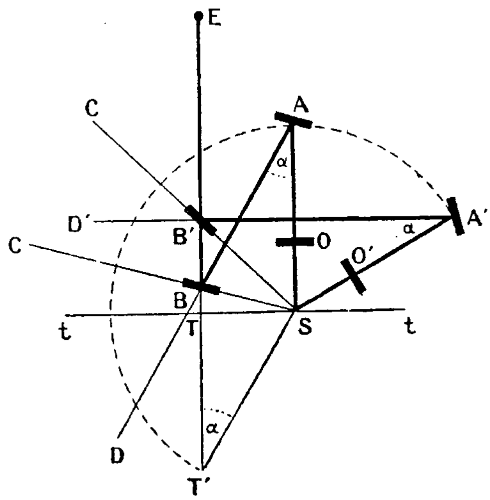

Fig. 6

In Fig. 6 the paper represents a plane containing the tilt axis *t-t* and the optical axis of the periscope. The eyepiece of the periscope is fixed at *E* and its optical axis cuts the tilt axis at right angles at a distance *ST* from the pinhole *S*. Movable along the line *ET* is a rotatable mirror *B* whose plane is restrained always to point towards the pinhole *S*, by a rod *SC* which is free to slide parallel to the plane of the mirror through the rotatable mounting of *B* but is pivoted round a normal axis passing through *S*. Also rotatable around the axis at *S* is a rod *SA* on which is mounted the objective *O* of the periscope and a mirror *A* inclined so that the angle of reflection of the optical axis of the objective is α. The distance *SA* is made equal to *ST* cosec α. Rigidly attached to *SA* and making an angle α to it is another rod *AD* which is made to slide through the movable and rotatable mounting of *B* so that when *B* is moved along *ET* the rod *DA* causes the rod *AS* and consequently the objective *O* to swing round the pinhole.

With this arrangement, for any positions of the mirror *B* and for any pointing of the optical axis of the objective *O* the path of the optical axis of the latter will be reflected into the path of the optical axis of the eyepiece and the length of the path between the pinhole *S* and the eye will be a constant (1).

(1) *Such a device has other uses. For instance, by employing a pair of optical linkages, one for each eye, the field of view of a compound stereoscope may be traversed from side to side by the optical axes of the binocular objectives without moving the eyes or the photographs, thus enabling the observer to fuse stereoscopically homologous rays without eyestrain.*

By employing a combination of two linkages for each eye the field of view of a stereoscope may be completely traversed by the optical axes of the binocular objectives both from side to side and up and down without moving the eyes or the photographs. Magnification of the field of view could then be introduced to any extent desirable. One other essential feature of the linkage is that the image, as seen by the observer, remains erect no matter in what direction the optical axis of the objective is pointing and this is accomplished without the use of a rotating "dove" prism.

This can be proved as follows. In Fig. 6 let T' be the point on EBT produced where the latter is cut by a circle of radius SA centred at S. Make SA equal ST cosec α, then the angle $BT'S$ must equal α. Now for any position of A such as A' the triangles $SA'B'$ and $ST'B'$ are similar, for by construction $B'S$ is common to both, SA' and ST' are equal and the angles $SA'B'$ and $ST'B'$ are equal. Thus the distance $EB' + B'A' + A'S$ must be constant because it always equals the constant distance $ET' + T'S$. Also, because the angles $T'B'S$ and $A'B'S$ are equal, then by the ordinary laws of reflection the ray from A' must be reflected at B' along the path $B'E$ (1).

In conclusion, it would seem reasonable to suggest that the principle of the pinhole mirror and real index mark should be applied to the construction of plotting instruments adapted for low obliques and verticals and in the construction of stereoscopic plotting and measuring instruments. It has the virtue of simplicity in that the accuracy of the plotting is not dependent on the precision of the mechanical movements which permit the viewing apparatus to scan the photographs, and this, it may be added, is not true of the stereoscopic plotting instruments at present in use.

It should also be noted that the pinhole mirror oblique plotter employs the reverse process to another type of photographic mapping instrument in which the image of the photograph is projected into a plane parallel to the map. In the latter type the optical problems of illumination and focus are more complicated than in the case of the pinhole mirror plotter, in that it is a question of projecting an inclined plane surface rather than one point into a plane of reference.

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(1) *A further interesting geometrical property of this device, which can be deduced, is that the speed of rotation of the arm* SA *about* S *is always twice that of the arm* SC.